

# Exercises on Homology and Cohomology

Spring term 2018, Sheet 5

Hand in before 10 o'clock on 26th March 2018  
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## Exercise 1 (easy)

Let  $(X, A)$  be a pair of spaces and  $H_*$  homology theory. Show that the following statements are equivalent:

- (i) the inclusion  $A \hookrightarrow X$  induces an isomorphism on all homology groups  $H_n(A) \rightarrow H_n(X)$ ,
- (ii)  $H_n(X, A) = 0$  for all  $n \in \mathbb{N}$ .

## Exercise 2 (medium)

Show that the relative singular homology group  $H_1(\mathbb{R}, \mathbb{Q})$  is free abelian and find an explicit basis.

## Exercise 3 (difficult)

Show that finite dimensional compact  $\Delta$ -complexes are automatically finite.