Exercises on Lie groups

Spring term 2018, Sheet 12

Hand in before 10 o'clock on 18th May 2018
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Exercise 1
Let $\mathfrak{g}$ be a finite dimenensional Lie algebra.

(i) Show that $\mathfrak{g}$ is nilpotent if and only its lower central series defined by

$$\mathfrak{g}_1 = \mathfrak{g}$$
$$\mathfrak{g}_{n+1} = [\mathfrak{g}_n, \mathfrak{g}]$$

terminates, that is there is some $n \in \mathbb{N}$ such that $\mathfrak{g}_n = 0$.

(ii) Show that every finite dimensional nilpotent Lie algebra is solvable.

Exercise 2
Let G be a connected Lie group. Show that G is nilpotent if and only if $\mathrm{Lie}(G)$ is nilpotent.

Exercise 3
Prove Lie's theorem: every solvable Lie algebra $\mathfrak{g} \leq \mathfrak{gl}(n, \mathbb{C})$ has a common eigenvector.

Exercise 4.
Show that the strictly upper triangular matrices are nilpotent and that they are equal to the derived Lie algebra of the upper triangular matrices.